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FROM: DTIC Classified Users Only. Controlling DoD Organization: Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD 21005. 22 Apr 1942.

AUTHORITY
EO 10501, 5 Nov 1953; 5 Nov 1953, per DoDI 5230.24

BALLISTIC RESEARCH LABORATORY

REPORT

THE RAPID PHOTOGRAMMETRY OF THE SPLASHES OF
BULLETS FIRED FROM AN AIRPLANE

BY

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ABERDEEN PROVING GROUND
ABERDEEN, MD.

April 22, 1942

Report No. 274

Aberdeen Proving Ground, Md.
April 28,1942.

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Abstract

In studying experimentally the trajectories of
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are photographed by a camera in the airplane. Tar-
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photographed. It is necessary to find the positions
of the splashes with respect to the airplane. Usual
photogrammetric procedures are very laborious. Here
a rapid method is described which makes use of the
altitude of the airplane, furnished by an altimeter.
The percentage error in the range to the splash is
one third of the percentage error of the altimeter
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In connection with the experimental studies, which the Lab-
oratory is now carrying out, of the trajectories of projectiles fired
from airplanes, bullets are fired from an airplane into water over
which the airplane is flying. It is necessary to find the range
from the splash of a bullet to the position of the gun at the instant
of the bullet's splashing. The splashes are photographed by a cinema
camera mounted on the airplane, and near to the gun.

If the focal length of the camera is known, and if the
coordinates on the water of three photographed targets are known, it
is clear that the space coordinates of the camera can be found, and
also the distance from the camera to a bullet-splash. The photogram-
metric computations, however, are so laborious that it is impractic-
able so to determine the range to each of a large number of splashes.
Likewise, if the coordinates of four or more photographed targets are
known, then the focal length, camera position, and range can be de-
termined. This solution also is very laborious. If the angle of
depression of the camera's axis could be measured once and for all at
the outset of a series of firings, then the range could easily be
found from the airplane's altitude, measured with an altimeter; but the
roll of the airplane, always present to some extent, causes the angle
of depression to vary from moment to moment so that ranges, determined

on the assumption of a constant angle of depression, are not accurate. The following rapid procedure has been devised to meet these difficulties.

In the diagram below, the origin, O, of the right-handed system of axes x,y,z is the principal point of the camera lens. Oz is perpendicular to the plane of the film or plate, Ox is horizontal, and Oy points above the horizon. Oz is depressed below the horizontal by the angle γ, to be determined, and intersects the surface of the water at a point O_w. The altitude of the camera above the water is h. The focal length of the camera is K, to be determined once and for all by any accurate method and recorded.
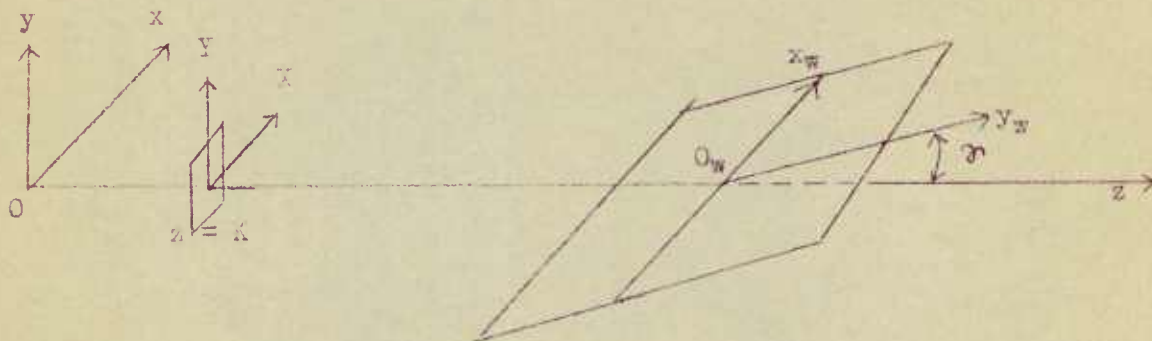
The image on the plate is inverted. After the plate has been erected, the coordinates (X,Y) of the image of any point P(x,y,z) are given by the intersection of OP with the plane z = K. The plate coordinates (X,Y) are measured from the "base" of the plate (the foot of the perpendicular from the nodal point to the plate), the X and Y axes being parallel to the x and y axes, respectively. For practical purposes, the "base" of the plate is the center of its exposed portion.

Let Q be a point on the erected plate, with plate coordinates (X,Y). The space coordinates of Q are then (X,Y,K), and thus Q is the image of points on the line

$$(x/X) = (y/Y) = (z/K).$$

The equation of the surface of the water is

$$y \cos \gamma - z \sin \gamma + h = 0.$$

The line meets the water at the point

$$x = (Xh)/(K \sin \gamma - Y \cos \gamma); \quad y = Yh/(K \sin \gamma - Y \cos \gamma);$$

$$z = Kh/(K \sin \gamma - Y \cos \gamma).$$

Let $O_w x_w$ and $O_w y_w$ be coordinate axes on the water's surface, such that the x_w axis is parallel to the x axis and the y_w axis is in the direction of increasing range. Then $x_w = x$, $y_w = y/\sin \gamma$. Hence

$$x_w = Xh/(K \sin \gamma - Y \cos \gamma); \quad y_w = Yh/(K \sin \gamma - Y \cos \gamma) \sin \gamma.$$

The ratio of any small element of area on the water to the area of its image on the plate, dW/dP, is given by

-5-

$$\frac{dW}{dP} = \frac{\partial(x_w, y_w)}{\partial(X, Y)}$$

$$= h^2 K/(K \sin \tau - Y \cos \tau)^3$$

by the theory of Jacobian determinants.

The area, W, on the water corresponding to any area P on the plate is therefore given by

$$W = \frac{h^2}{K^2(\sin \tau - (\overline{Y}/K)\cos \tau)^3} \iint \frac{dP}{(1-ay'')^3}$$

where $\overline{Y}$ is the Y coordinate of the centroid of the area on the plate, where $y'' = Y - \overline{Y}$, and where

$$a = \cos \tau / (K \sin \tau - \overline{Y} \cos \tau).$$

The integrand may be expanded thus:

$$1 + 3ay'' + 6a^2 y''^2 + \ldots$$

and the integral of the first term is P, the integral of the second term is zero, and the integral of the third term is $6a^2 P$ times the average value, $\overline{y''^2}$, of y''^2 over the area P. Thus

$$\sin \tau - (\overline{Y}/K) \cos \tau = \left(\frac{h^2 P}{K^2 W}\right)^{1/3} (1 + 2a^2 \overline{y''^2} + \ldots). \qquad (1)$$

The term $2a^2 \overline{y''^2}$ is no larger than 0.01 for a camera whose vertical field is ten degrees, and which is depressed below the horizontal by an angle of twenty-two degrees, provided that the area P extends over only one half the height of the plate in the Y direction. For smaller areas, and greater angles of depression, the term is still smaller. Provided that an error of one per cent in the range to a splash can be tolerated, then for a camera of the type described the term can be ignored altogether.

The areas P and W may be found readily from the coordinates of the images of three targets on the plate, and from the coordinates of the targets on the water. If $P_1(x_1, y_1)$, $P_2(x_2 y_2)$, and $P_3(x_3, y_3)$ are any three points in a plane, then the area of the triangle $P_1 P_2 P_3$ is

$$A = \frac{1}{2} \begin{vmatrix} x_2-x_1, & y_2-y_1 \\ x_3-x_1, & y_3-y_1 \end{vmatrix} \qquad (2)$$

where the determinant is given its absolute value. For the same three points, the same area A is obtained however the subscripts are assigned to the points. Moreover, the area A is invariant with respect to rotations and translations of the axes of x and y. Thus formula (2) will furnish the area W of the triangle formed by three targets, from the coordinates of the targets in the system in which they have been surveyed. Likewise, applied to the measured coordinates of the images of the three targets on the plate, the formula (2) will provide the area P even though the axes used for measurement of the images differ from the axes X,Y in direction and origin.

The range to a particular splash in the picture is

$$R = \frac{h}{\sin\tau - (Y/K)\cos\tau}\left(1 + \frac{X^2 + Y^2}{K^2}\right)^{\frac{1}{2}} \qquad . \qquad (3)$$

Here (X,Y) are the plate coordinates of the splash in the system of axes X,Y. In (3), the quantity

$$\left(1 + \frac{X^2 + Y^2}{K^2}\right)^{\frac{1}{2}}$$

may be set equal to unity for a camera having a field 14° x 10°, with a resulting error in the range of no more than about one per cent. Further, the value of R given by (3) is insensitive to the value of Y in the denominator, and hence this Y can be obtained with sufficient accuracy in practice (for a camera of the type described) from a rough knowledge of the direction, through the center of the picture, of the X axis--a horizontal axis. In the same way, $\overline{Y}$ can be found with sufficient precision. To find Y and $\overline{Y}$, it is desirable to measure the plate in terms of axes α, β such that the α axis is approximately horizontal. Then Y is merely the β of the splash diminished by the β of the center of the plate, and $\overline{Y}$ is merely the average of the β's of the three targets, diminished by the β of the plate center.

It will be noticed from (1) that $\sin\tau$ varies roughly as $h^{2/3}$, so that by (3) the range varies as the cube root of h. Thus an error of three per cent in h introduces an error of only one per cent in R. The preceding method is thus convenient for finding the range, with an error whose average value has been found (for a camera of the type described) to be about two per cent.

Example. In the following example, the photograph was enlarged before measurement. The units of α, β are ten inches; the equivalent focal length of the enlarged image is 12.56 units of ten inches. The origin of α, β is the upper right-hand corner of the plate. The center of the plate is at $\beta = 1.161$, and α and β are measured to the left and down from the origin, respectively, the α axis being nearly horizontal. The water coordinates, x,y, are in units of 100 feet; the altimeter altitude (corrected for air speed) is 260 feet. The splash to which the range is desired is at $\beta = 0.962$. The work-sheet follows (some numbers, retained on the computing machine, not being entered).

Target	α	β	α'	β'	x	y	x'	y'	
5	2.365	.955	2.115	.242	1.769	1.302	---	---	h = 2.6
4	1.705	.406	1.455	.791	1.983	2.156	.214	.854	$\sin\tau = .4256$
8	.250	1.197	---	---	2.778	1.295	1.009	-.007	$\cos\tau = .9050$
mean		.853							R = 632 feet
$\overline{Y} = .308$					2P = 1.3209				
$Y_{splash} = .199$				$\sin\tau - .0245$	$\cos\tau = .4033$	2W = 0.8632			

The areas are computed by equation (2), $\overline{Y}$ is found from the mean β, and equation (1) is written down(ignoring the term $2a^2y''^2$) and solved by successive approximations. It is solved more rapidly by successive approximations than it could be solved as a quadratic equation. From the Y of the splash, found from it β, the range R to the splash is computed from equation (3), in which the quantity $(1 + (X^2+Y^2)/R^2)$ is set equal to unity.

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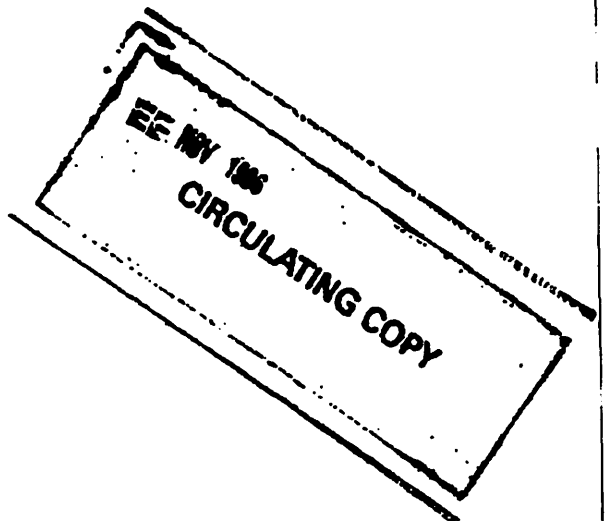
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